

Week 7 Introduction

Plan:

1. Background: Where we have gotten to on understanding relations between discursive practices and reason relations.

a) Two paired, parallel notions of implicitness:

i. Implicit *commitment* (on side of practices):

$\Gamma|\sim A$ read as:

Commitment to accept all of Γ precludes entitlement to deny A (or *all* of Δ , for $\Gamma|\sim\Delta$).

Preclusion of *entitlement* to *deny* A is *implicit* commitment to *accept* A.

It differs from the *explicit* acknowledgment of commitment to accept the elements of Γ that consists, in the first instance, in actually accepting them.

There are niceties here concerning just how

- practical attitudes of accepting,
- public speech acts of asserting,
- normative statuses of commitment to accept, and
- acknowledgment of commitment to accept

are related.

Here are some guideposts:

Asserting:

In the most basic cases, **one overtly accepts a claimable by asserting it.**

(That is why Restall and Ripley use public speech acts of assertion/denial, rather than attitudes of acceptance/rejection as the subjects of normative assessment as “out of bounds” or not.)

What one *accepts* (respectively *rejects*) is what one is *explicitly* committed to (accept/reject).

Conceptually, acceptance begins with assertion (rejection with denial).

But then (see below), we can make sense of these attitudes unaccompanied by overt speech acts expressing them.

In *asserting* the *attitude* of acceptance and the *normative status* of commitment to accept coincide.

In assertion one *explicitly* acknowledges commitment to accept by fulfilling that commitment, namely by accepting the claimable.

Assertion is the public *undertaking* of the commitment to accept, by *being* the acceptance (Correspondingly for speech acts of denial and practical attitudes of rejection.)

Assertion is the overt adoption of a practical *attitude* that *institutes* the *normative status* in question.

What makes the commitment *explicit* is the attitude of acknowledging it that consists in actually accepting (or rejecting, paradigmatically by denial) the claimable.

Accepting:

Against the background of practices of publicly accepting/rejecting in assertion/denial, we can then make sense of the adoption of those attitudes—the acknowledgment of what count as *explicit* commitments just in virtue of being actually acknowledged by the adoption of those attitudes—without the performance of overt speech acts.

Here the image is of assertion *in foro interno* (as Sellars says).

Less fancifully, it is a practical attitude that involves a disposition to assert, publicly to acknowledge that explicit commitment, if suitably prompted.

(Here a *ceteris paribus* clause is in order, used properly, to acknowledge the defeasibility of the implication I am asserting.)

ii. *Implicit content* (on side of reason relations):

$\Gamma \sim A$ involves two sorts of content for the set of sentences Γ .

- The sentences that are *elements* of the set Γ are what it *explicitly contains*, articulating its *explicit content*.
- The consequences of Γ , sentences A that it *implies* are what it *implicitly contains*, articulating its *implicit content*.

b) Two paired *processes* of moving from the explicit to the implicit, on the *pragmatic* dimension of explicit/implicit *commitment* and on the *rational* dimension of explicit/implicit *content*:

i. On the side of reason *relations* we introduced the notion of *rational explicitation*: Making *implicit* content *explicit*.

This is considering the difference in *implicit* content between a premise-set Γ , when $\Gamma \sim A$, and the new premise-set Γ, A that *explicitly* contains some claimable that Γ contains only *implicitly*.

Here the key claims are:

- The structural principle CM (Cautious Monotonicity) says that explicitation never *loses* implicit content:
Anything that Γ implies, $\Gamma \cup \{A\}$ still implies.
CM entails $\{X: \Gamma | \sim X\} \subseteq \{Y: \Gamma, A | \sim Y\}$. Explicitation does not *subtract* consequences.
- The structural principle CT (Cumulative Transitivity) says that explicitation never *gains* implicit content:
Anything that $\Gamma \cup \{A\}$ implies was already implied by Γ .
CT entails $\{Y: \Gamma, A | \sim Y\} \subseteq \{X: \Gamma | \sim X\}$. Explicitation does not *add* consequences.
- Together, CM and CT entail that *explicitation is inconsequential*: making implicit content explicit does not affect the remaining implicit content in any way.
- Though no doubt explicitation *often* is inconsequential, it is not so in general, that is, *always*. Sometimes explicitation has important consequences for the content of the resulting premise-set. (I offered a database + inference engine model, and an observation/theory model to argue this.)

[So far, recap. Here is the new bit:]

- ii. On the side of *discursive practices*, which are *normatively governed* by reason relations, we consider the *process* of turning *implicit* commitments into *explicit* commitments.

That is what one *does* when one *explicitly acknowledges* something that, before that *act*, one was only *implicitly* committed to: *explicitly accepts* what one had been *implicitly* committed to accept.

Since one is implicitly committed to accept just what one is precluded from being *entitled* to *reject*, this is not the same as *explicit* acceptance.

I now suggest that **what one is *doing* in changing the pragmatic status of commitment to accept a claimable content from being something one is only *implicitly* committed to accept to being something one actually accepts—the practical *acknowledgement* of that implicit commitment—is *inferring*.**

Inferring is just explicitly acknowledging commitment to accept what one is implicitly committed to accept, by *accepting* that claimable content.

This might be done *overtly*, by *asserting* it, performing that speech act, or it might be done only *covertly*, by a change in practical attitude, by *accepting* that content.

In either case, inferring is a substantial move.

It is a significant alteration of status.

It puts the inferrer in a new discursive normative pragmatic situation.

Inference can be *ampliative*, yielding new implicit commitments.

And inference can be *reductive*, removing old implicit commitments.

It is *the* paradigmatic rational activity: coming explicitly to *realize* and *accept* what one was hitherto only *implicitly* committed to accept.

c) So here is the point:

- Just as we can line up notions of explicit and implicit *commitment*, on the pragmatic side of discursive practices with notions of explicit and implicit *content*, on the rational side of reason relations, so too we can line up the practical activity of *inferring* (explicitly acknowledging implicit commitments) with the rational activity of *explicitation* (making explicit what was otherwise implicit content).
- And treating *rational* explicitation as *inconsequential*, as the structural principles of CM and CT would require us to do, would preclude us from understanding how *inference* can make a substantive normative difference, how it can add new implicit commitments and subtract old ones: how it can be **ampliative** and lead to new knowledge or **corrective** and guard us from old mistakes.
- **If explicitation is inconsequential, then inference is impotent.**
But: inference is not impotent, so explicitation is not inconsequential.
- So CM and CT should not be imposed globally.
- So we want logical metavocabularies to be able to codify reason relations with *open* structure (nonmonotonic and nontransitive).

Recall that last time I connected the consequentiality of explicitation to the phenomenon of *rational hysteresis*: the path-dependence of the extraction of consequences, as a result of which where one ends up—what follows from the premises one started with, what their implicit content turns out to be—depends essentially on the *order* in which you extract consequences.

We can now see this as a feature of the *process of inference*.

The important thing to realize is that this path-dependence of the drawing of consequences is not a *psychological* matter.

It is a feature of the reason relations themselves.

It infects the very notion of the implicit content of the original premise-set.

This phenomenon is the origin of the *intrinsic historicity of reason* (relations), manifested in the historicity of reasoning practices, specifically rationality as manifested in *inferring*.

2. Last time:

- a) I introduced a technical notion of a vocabulary. A vocabulary is a *lexicon*, L a set of sentences, together with a set R^2 of *reason relations* on that lexicon. $\langle L, R^2 \rangle$.

b) We saw how we could, largely* without loss of generality represent the reason relations by sets of pairs of sets of sentences. $\langle \Gamma, \Delta \rangle \in R^2$ iff $\Gamma, \Delta \subseteq L$ and $\Gamma \mid \sim \Delta$. Incoherent sets (and thereby, relations of incompatibility, are marked by implications with empty right-hand sides, so that Γ is incoherent iff $\langle \Gamma, \emptyset \rangle \in R^2$.

(*) only “largely” because monotonicity structures of implication will be inherited by incompatibility if we use this notational convenience. Imposing MO, or CM, on implication will impose the same structural condition on incompatibility.

c) I introduced a way of thinking about *logical* vocabularies.

These are *rational metavocabularies*, in the sense that the *lexicon* and *reason relations* of the *logical* metavocabulary is *wholly determined* in a systematic way by the lexicon and reason relations of some *base* vocabulary.

Further, logical vocabularies are distinguished from other rational metavocabularies (for instance, semantic ones), by being *conservative extensions* of their base vocabularies:

The lexicon of the logical metavocabulary includes the lexicon of the base vocabulary.

And the reason relations of the logical metavocabulary includes that of the base vocabulary, and does not change it: all the implications and incompatibilities of the logically extended vocabulary that are restricted to base vocabulary are the same as those of the base vocabulary.

In short: the logical vocabulary is a *conservatively extended elaboration* of the base vocabulary.

d) I offered a two-part characterization of the expressive role distinctive of logical vocabulary as such: it is **LX** for its base vocabulary. “LX” is short for “elaborated from and explicative of.”

I have just said, in (c), what I mean by the “L”: conservatively extended elaboration.

I gave two suggestive examples of how the “X” that requires logical vocabulary to be “explicative of” base vocabularies can be satisfied:

The conditions on conditionals as making explicit (putting into sentential form) *implication* relations, and negation as making explicit *incompatibility* relations:

Deduction-Detachment (DD) Condition on Conditionals: $\Gamma \mid \sim A \rightarrow B$ iff $\Gamma, A \mid \sim B$.

Incoherence-Incompatibility (II) Condition on Negation: $\Gamma \mid \sim \neg A$ iff $\Gamma \# A$.

e) In addition, I articulated the expressive role characteristic of *logical* rational metavocabularies as being **universally and comprehensively LX**.

To say that they are *universally* LX is to say that they can be elaborated from and explicative of *any* and *all* vocabularies.

To say that they are *comprehensively* LX is to say that they explicate the reason relations not only of the *base* vocabulary, but also of the *logically extended* vocabulary that is LX for that base.

- f) We looked at how a base lexicon can be elaborated and conservatively extended into a lexicon consisting of logical compounds of the base lexicon, by *closing* it under *formation rules*.

L is the smallest (by inclusion) superset of L_0 such that if the elements above the line are in L, then so are the elements below the line:

$$\frac{\alpha, \beta \in L}{\neg\alpha \in L \quad \alpha \rightarrow \beta \in L \quad \alpha \& \beta \in L \quad \alpha \vee \beta \in L.}$$

- g) To extend the *reason relations* R^2_0 of the base vocabulary, we do something of the very same form. This time, the rules we close under are *connective* rules:

Corresponding to the requirement that $L_0 \subseteq L$, we have:

Axiom of NM-MS:

$$\frac{\Gamma \sim_0 \Delta}{\Gamma \sim \Delta}$$

And corresponding to the rules we close under are rules of the form:

$$\text{L\&:} \quad \frac{\Gamma, A, B \sim \Theta}{\Gamma, A \& B \sim \Theta} \qquad \text{R\&:} \quad \frac{\Gamma \sim A, \Theta \quad \Gamma \sim B, \Theta}{\Gamma \sim A \& B, \Theta}$$

R^2 is then the smallest superset of R^2_0 such that if the sequent (implication, reason relation) above the line is in R^2 , then so are the sequents below the line.

- h) We just do that for *all* the rules of NM-MS:

Expressive (Principal) Connectives:

$$\begin{array}{ll} \text{L}\rightarrow: & \frac{\Gamma \sim \Theta, A \quad B, \Gamma \sim \Theta}{A \rightarrow B, \Gamma \sim \Theta} \qquad \text{R}\rightarrow: & \frac{A, \Gamma \sim \Theta, B}{\Gamma \sim \Theta, A \rightarrow B} \\ \text{L}\neg: & \frac{\Gamma \sim A, \Theta}{\neg A, \Gamma \sim \Theta} \qquad \text{R}\neg: & \frac{A, \Gamma \sim \Theta}{\Gamma \sim \neg A, \Theta} \end{array}$$

Aggregative (Auxiliary) Connectives:

$$\begin{array}{ll} \text{L\&:} & \frac{\Gamma, A, B \sim \Theta}{\Gamma, A \& B \sim \Theta} \qquad \text{R\&:} & \frac{\Gamma \sim A, \Theta \quad \Gamma \sim B, \Theta}{\Gamma \sim A \& B, \Theta} \\ \text{L}\vee: & \frac{A, \Gamma \sim \Theta \quad B, \Gamma \sim \Theta}{A \vee B, \Gamma \sim \Theta} \qquad \text{R}\vee: & \frac{\Gamma \sim A, B, \Theta}{\Gamma \sim A \vee B, \Theta} \end{array}$$

3. Dan will now explain explication by NM-MS in detail.

Two different ways:

- a) His expressive completeness representation theorem.
- b) Codifying local regions of structure: monotonicity (MO) and classicality (CO).

Extra Material [that I probably won't get to]:

Conditional:

The conditional on the right, $R \rightarrow$, is just the Deduction-Detachment, Dual Ramsey conditional

To perform its defining expressive task of codifying implication relations in the object language, conditionals need to satisfy the

Ramsey Condition: $\Gamma \sim A \rightarrow B$ iff $\Gamma, A \sim B$.

That is, a premise-set implies a conditional just in case the result of adding the antecedent to that premise-set implies the consequent.

(A conditional that satisfies this equivalence can be called a "Ramsey-test conditional," since Frank Ramsey first proposed thinking of conditionals this way.)

All these Ketonen connective rules are *reversible*.

This will be important for Dan Kaplan's *expressive completeness* result for NM-MS.

$L \rightarrow$ takes some thinking about.

It is just a shared-context version of Gentzen's LK left-rule for the conditional.

(His mixed context version builds in monotonicity, which basically just is context mixing.)

The comma on the right is disjunctive.

This is another manifestation of bilateralism: reading the two sides of the turnstile differently.

If denying everything on the right of the turnstile is out of bounds (the RR-bilateralist reading of the turnstile), then one is implicitly committed to accepting *something* on the right.

That is disjunctive.

So the first premise of $L \rightarrow$ says that Γ implies Θ or A .

The second premise says that Γ together with B implies the conclusion Θ .

When those both hold, the $L \rightarrow$ rule says that Γ together with something, the conditional, that, given A gives you B is sufficient for the conclusion Θ .

If A is not true, then by the first premise Γ is sufficient for Θ .

If A is true, then the conditional will yield B (in the context of Γ), which the second premise says is enough, in that context, to get Θ .

Deduction-Detachment (Dual Ramsey) principle only gives us a rationale for the right rule for the conditional: when it is implied as a consequence.

But we can turn that into a rationale for the left rule, describing its role as a premise.

For we would like the conditional rules to *preserve* CO, should the base vocabulary satisfy that absolutely minimal monotonicity principle.

That means that it should follow from the connective rules that

$\Gamma, A \rightarrow B \sim A \rightarrow B$. We know by $R \rightarrow$ that that will hold iff $\Gamma, A \rightarrow B, A \sim B$.

That all *such* sequents hold is just detachment on the left of the turnstile.

And $L \rightarrow$ is just what is needed to guarantee that, supposing the base satisfies CO.

$L \rightarrow$ (which, recall, is reversible) says that to get $\Gamma, A, A \rightarrow B \vdash B$, we need two premises:
 $\Gamma, A \vdash B, A$ and $\Gamma, A, B \vdash B$.
 But both of these are instances of CO, so we always have them.
 (In $L \rightarrow$, Θ is in our example, B , and (confusingly) Γ is Γ, A .)

Conjunction:

Compare (LK):

$$L\&(\text{additive}): \quad \frac{\Gamma, A \vdash \Theta}{\Gamma, A \& B \vdash \Theta} \quad \frac{\Gamma, B \vdash \Theta}{\Gamma, A \& B \vdash \Theta}$$

This standard classical definition builds in monotonicity on the left.

It says that if, in the context Γ , A implies something, Θ , then it does so no matter what we conjoin with it. That is, the original implication has no defeaters.

So we require instead that to get the conjunction, we need that A *together with* B imply Θ .

In LK, Gentzen used Mixed Context versions of $L \rightarrow, L \vee, R \&$

Negation:

For negation, think of $R \neg$ but with Θ empty, signifying incoherence, like \perp .

$$\text{Then } R \neg \text{ becomes: } \frac{A, \Gamma \vdash \perp}{\Gamma \vdash \neg A}$$

This is just the

Incoherence-Incompatibility Negation Condition: $\Gamma \vdash \neg A$ iff $\Gamma, A \vdash \perp$.

Anything incompatible with A implies $\neg A$.

It follows that $\neg A$ is the minimal incompatible of A , in the sense of being implied by everything that is incompatible with A .

$$L \neg \text{ becomes: } \frac{\Gamma \vdash \neg A}{\Gamma, \neg A \vdash \perp}$$

If Γ implies A , then Γ is incompatible with $\neg A$.

I. Combining Supraclassicality and Nonmonotonicity

Dan shows that if NM-MS is applied to all and only base vocabularies that consist entirely of instances of CO (a “flat” prior), then the result is just classical logic.

More than this, NM-MS is supraclassical. But, not only is its consequence relation—the consequence relation (and incompatibility-incoherence) of the logically extended base vocabulary—nonmonotonic, so is its *purely logical* consequence relation: the consequences that hold *purely* in virtue of logic (the connective definitions) alone.

There are two things one can mean by “the *purely logical* reason relations of NM-MS,” once one has realized that these are *not* to be identified with the reason relations over the logically extended lexicon for any particular base vocabulary.

- i) Local: Within any such specific NM-MS extension of a particular base vocabulary, the consequences that are good and remain good on arbitrary replacement of non-logical with non-logical vocabulary.
- ii) Global: The consequences involving logically complex sentences that hold good no matter what base vocabulary we apply NM-MS to.

These two do *not* in general coincide.

To show how supraclassicality works.

NM-MS is supraclassical w/res to *theorems*, in that every theorem of LK is a theorem of NM-MS.

NM-MS is supraclassical w/res to *purely logical consequence relations*. Every *purely logical* metainference licensed by LK is licensed by NM-MS

NM-MS is *not* supraclassical w/res to the *consequence relation* of the logically extended vocabulary resulting from applying NM-MS rules to any non-flat base vocabulary.

Suppose the base globally satisfies CO.

(Dan will show us what to do if only *some* regions of base have CO holding, but we don’t care about that now.)

Global CO is preserved for logically extended language.

All classical theorems will then be consequences of every premise-set.

That means that for all premise-sets Γ : $\Gamma \mid \sim (A \rightarrow C) \rightarrow ((A \& B) \rightarrow C)$.

How can that be so without forcing monotonicity?

Doesn’t it say that no matter what Γ is, if in that context an implication $A \mid \sim C$ holds then in that same context $A \& B \mid \sim C$ holds, so $A, B \mid \sim C$ holds? (No.)

Monotonicity would be if there were a good argument of the form

$\Gamma, A \mid \sim C$

$\Gamma, A, B \mid \sim C$.

How is that not licensed when $\Gamma \mid \sim (A \rightarrow C) \rightarrow ((A \& B) \rightarrow C)$?

The key is that *modus ponens* holds on the left of the turnstile (for premises), but not on the right (for conclusions). To play its expressive role (dual Ramsey, DD) it must hold, as it were, *across* the turnstile. But that is compatible with its holding on the left but not the right.

Claim 1: Can detach from conditionals on the left.

This is when both the conditional (C) and the antecedent (A) are *explicit*: CEAE.

$\Gamma, A, A \rightarrow B \mid \sim B$.

$L \rightarrow$ (which, recall, is reversible) says that to get $\Gamma, A, A \rightarrow B \mid \sim B$, we need two premises: (In $L \rightarrow$, Θ is in our example, B, and (confusingly) Γ is Γ, A .)

$\Gamma, A \mid \sim B, A$ and $\Gamma, A, B \mid \sim B$.

But both of these are instances of CO, so we always have them.

$\Gamma, A \mid \sim B, A$ and $\Gamma, A, B \mid \sim B$
 $\Gamma, A, A \rightarrow B \mid \sim B$.

Claim 2: Cannot detach from conditionals on the right.

This is when both the conditional (C) and the antecedent (A) are *implicit*: CIAI*.

We cannot argue from $\Gamma \mid \sim A$ and $\Gamma \mid \sim A \rightarrow B$ to $\Gamma \mid \sim B$.

That is: **NOT:**
$$\frac{\Gamma \mid \sim A \quad \Gamma \mid \sim A \rightarrow B}{\Gamma \mid \sim B}$$

Why not? $\Gamma \mid \sim A \rightarrow B$ iff (reversible $R \rightarrow$) $\Gamma, A \mid \sim B$.

That is just the Dual Ramsey, Deduction-Detachment condition, in virtue of which the conditional \rightarrow expresses (codifies, makes explicit in sentences in the logically extended language) implications, $\mid \sim$.

So the argument we are assessing can be rewritten as:

$$\frac{\Gamma \mid \sim A \quad \Gamma, A \mid \sim B}{\Gamma \mid \sim B}$$

That is just Cut (CT).

Gentzen imposes that globally.

But we do not.

That transitivity principle says that explicitating the implicit (implied) A does not *add* any consequences.

Without that transitivity principle, we can't detach from an implied conditional with a merely implied antecedent. Only an *explicit* conditional and antecedent is detachable.

Claim 3: Cannot detach an explicit conditional with an implicit (implied) antecedent: CEAI*.

What about $\Gamma|\sim A$ and $\Gamma, A \rightarrow B|\sim C$? Can we get $\Gamma|\sim C$? No.

If $\Gamma, A \rightarrow B|\sim C$, we know by $L \rightarrow$ that $\Gamma|\sim C, A$ and $\Gamma, B|\sim C$.

We also have $\Gamma|\sim A$.

So the argument at issue is:

$$\frac{\Gamma|\sim A \quad \Gamma|\sim C, A \quad \Gamma, B|\sim C}{\Gamma|\sim C} \quad ?$$

The third premise is irrelevant, since B isn't in the conclusion or the other premises.

But what we can get from $\Gamma|\sim C, A$ is $\Gamma, \neg A|\sim C$, and $\Gamma|\sim A$ doesn't help getting $\Gamma|\sim C$.

There are two different things one could mean by CEAI:

$$1. \quad \frac{\Gamma|\sim A}{\Gamma, A \rightarrow B|\sim B}$$

By $L \rightarrow$, $\Gamma, A \rightarrow B|\sim B$ must come from $\Gamma|\sim B, A$ and $\Gamma, B|\sim B$.

The latter holds because of CO.

But $\Gamma|\sim B, A$ only follows from $\Gamma|\sim A$ by MO on the right. And that does not hold.

So there is no way to get $\Gamma, A \rightarrow B|\sim B$ from $\Gamma|\sim A$.

2. But if we have instead $\Gamma, A \rightarrow B|\sim A$ as the “implicit antecedent” premise, then $L \rightarrow$ says this has to come from $\Gamma|\sim A, A$, which is $\Gamma|\sim A$, and $\Gamma, B|\sim A$.

This last would only follow from $\Gamma|\sim A$ by MO on the left, which does not hold, and would only follow from that plus $\Gamma, A \rightarrow B|\sim A$ if we could detach in the CEAI case of explicit conditionals and implicit antecedents. [We just showed in Claim 3 that this does *not* hold.

So CIAE does not hold either. No we didn't.]

Claim 4: Can detach from implicit Conditional, explicit antecedent CIAE:

$$\frac{\Gamma, A|\sim A \rightarrow B}{\Gamma, A|\sim B}$$

By $R \rightarrow$, $\Gamma, A|\sim A \rightarrow B$ hold just in case $\Gamma, A, A|\sim B$.

But if that holds, then so does $\Gamma, A|\sim A \rightarrow B$, since we are using sets (so accepting Gentzen's Contraction).

Summary: Two of the four possible detachment conditions hold universally for the vocabularies that result from applying NM-MS to base vocabularies that satisfy CO (and perhaps a lot else): CEAE and CIAE—the two that have the antecedent explicitly. In that case, the conditional on *either* side of the turnstile suffices for the implication of the consequent of the conditional.

Analysis: This is a signal case of rational *explication* being consequential, and of *inference* making a difference to *implicit* commitments.

Merely being committed to premises that *imply* a conclusion is not enough to commit one, even implicitly, to the consequents of conditionals with that antecedent—whether or not one is committed to the conditional explicitly, or only implicitly.

However, if one is *explicitly* committed to the antecedent, then either explicit or implicit commitment to the conditional implicitly commits one to the consequent.

With these 4 claims about detachment settled, let's look again at how and why

$\Gamma \mid \sim (A \rightarrow C) \rightarrow ((A \& B) \rightarrow C)$

fails to license the monotonicity move:

$$\frac{\Gamma, A \mid \sim C}{\Gamma, A, B \mid \sim C}.$$

$R \rightarrow$ tells us that $\Gamma \mid \sim (A \rightarrow C) \rightarrow ((A \& B) \rightarrow C)$ must have come from $\Gamma, A \rightarrow C \mid \sim (A \& B) \rightarrow C$.

That in turn comes from $\Gamma, A \rightarrow C, A \& B \mid \sim C$.

Unpacking that by L&, we need to have $\Gamma, A \rightarrow C, A, B \mid \sim C$.

But that we get by detachment on the left.

That is why $\Gamma \mid \sim (A \rightarrow C) \rightarrow ((A \& B) \rightarrow C)$ for every Γ .

The implication $\Gamma, A, A \rightarrow C \mid \sim C$ can be weakened by arbitrary B.

But that does not at all imply that $\Gamma, A \mid \sim C$ can be weakened by arbitrary B, which is the monotonicity move.

It is essential to the goodness of $\Gamma, A, A \rightarrow C, B \mid \sim C$ that the conditional is in the antecedent.

[Really stray notes from here out.]

We can also look in a bit more detail at incoherence, incompatibility, and negation.

If Γ is incoherent, then $\Gamma \mid \sim$.

Then for any $A \in \Gamma$, $\Gamma \mid \sim \neg A$.

Γ is *explicitly* incoherent.

But it need not be *persistently* incoherent.

There can certainly be B s.t. Γ, B is not incoherent.

There can even be B s.t. $\Gamma \mid \sim B$ s.t. Γ, B is not incoherent.

But if for any $A \in \Gamma$ s.t. $\Gamma \mid \sim \neg A$, on *explicitates* that negated consequence, the resulting premise set, $\Gamma, \neg A$ will contain both A (by hypothesis) and $\neg A$.

That is *persistently* incoherent.

We also allow base vocabularies (and *so* logically extended vocabularies) to be *materially paraconsistent*. That is, incoherent premise-sets need not imply everything. Explosion.

Graham Priest defines “paraconsistency” as denial of explosion.

He is thinking about the case where A and $\neg A$ are both in a premise-set, or sometimes, when A and $\neg A$ are both *implied by* a premise set.

In the former case, we *do* have explosion (of *persistently* incoherent premise-sets).

In the latter case, where the *inconsistency* (not merely *incoherence*) is only *implicit*, we do *not* have explosion.

But we can have an implicitly incoherent set that does not explode—where we can still reason with the premise-set, because there is a significant distinction between what it *does* imply and what it does *not* imply.

This is why we want to avoid explosion, and so have *some* variety of paraconsistency.

An explicitly incoherent set can fare two ways under explicitation.

As remarked above, if any of the negations of premises that it implies are explicitated, it becomes not just explicitly incoherent, but explicitly *persistently* incoherent, and so explodes implicationally.

But that very same set can have *other* consequences that are *not* negations of its premises.

Explicitating *them* might yield a *different* explicitly incoherent premise set.

Or it might *cure* the incoherence, and yield an explicitly *coherent* set.

Which might still be *implicitly* incoherent.

Or not. It might be implicitly coherent.

Since we stipulate that the whole lexicon must be incoherent—to guarantee that there are *some* incompatibilities—*no* premise set is *persistently coherent*. Every one has *some* incoherent superset, namely the whole lexicon.

LP has exactly the theorems of classical logic, differing only in the consequence relation.

So does NM-MS, when applied to a “flat” base vocabulary (a “flat prior”), consisting only of instances of CO.

[So I should expound Dummett on individuating logics by consequences and not theorems.]

By contrast, the tradition Frege initiated in the 1890's makes truth, rather than inference, primary in the order of explanation. Dummett says of this shift:

3] ...in this respect (and [Dummett implausibly but endearingly hastens to add] in this respect alone) Frege's new approach to logic was retrograde. He characterized logic by saying that, while all sciences have truth as their goal, in logic truth is not merely the goal, but the object of study. The traditional answer to the question what is the subject-matter of logic is, however, that it is, not truth, but inference, or, more properly, the relation of logical consequence. This was the

received opinion all through the doldrums of logic, until the subject was revitalized by Frege; and it is, surely, the correct view.¹

And again:

4] It remains that the representation of logic as concerned with a characteristic of sentences, truth, rather than of transitions from sentences to sentences, had highly deleterious effects both in logic and in philosophy. In philosophy it led to a concentration on logical truth and its generalization, analytic truth, as the problematic notions, rather than on the notion of a statement's being a deductive consequence of other statements, and hence to solutions involving a distinction between two supposedly utterly different kinds of truth, analytic truth and contingent truth, which would have appeared preposterous and irrelevant if the central problem had from the start been taken to be that of the character of the relation of deductive consequence.²

The consequence relation is nonexplosive.

That is what “**paraconsistent**” means.

It is not to be confused with “**dialethic**,” which means taking some contradictions to be true. (**We are paraconsistent, but not dialethic.**)

Premise-sets that *do* imply everything are the bad ones.

But there are a number of senses in which this can be true: implicit/explicit, persistent/contingent.

At the end, might want to tell the story, suggest the advantages of NM-MS, in context of database (set of premises) + inference engine (calculating reason relations).

Each time you change elements of database (premise-set), must recalculate its consequences and incompatibilities. But if one extends the language logically and extracts those consequences off

¹ Dummett, *Frege's Philosophy of Language* [Harper & Row 1973] (hereafter *FPL*), p. 432.

² Dummett, *FPL*, p. 433. A few comments on this passage: First, the “deleterious effects in logic” Dummett has in mind include taking logics to be individuated by their theorems rather than their consequence relations. Although one can do things either way for classical logic, in more interesting cases logics can have the same theorems but different consequence relations.

line, then it is easy to tell what would happen in the vicinity of the database if one, say, added A to it. Just look at the conditionals with A as antecedent. This could minimize online processing.